MEMORANDUM

Date:	September 25, 2017
To:	Poul Rosen, City of Nanaimo
cc:	Glen Shkurhan, Urban Systems
From:	Taylor Swailes, Glen Zachary
File:	1298.0048.02
Subject:	Chase River Regional Analysis

1. Overview

1.1 Introduction

This memo is provided in support of the memo “Colliery Middle Dam Hydraulic Assessment - FINAL” that Urban Systems completed March 21, 2017 for the City of Nanaimo.

The March 21, 2017 memo provided an assessment of the predicted design flows for the middle dam by using a continuous simulation PCSWMM model, calibrated to several years of observed data. The resulting model demonstrated a very good fit to observed data, but recognized that the largest event observed had a return period of approximately 1:2 years. Therefore, the predictions for extreme events were based on limited data. As such, that memo recommended further monitoring and analysis be undertaken over time.

We understand that the Ministry of Forests, Lands, and Natural Resource Operations (MFLNRO) has specifically asked the City to provide a regional analysis as a second method of establishing design flows to compare with the modeled results. This memo outlines the regional analysis method used, and the resulting peak flow rates generated, for Chase River.

The regional analysis method selected was the “index flood method” as outlined in the Canadian Dam Association (CDA) Technical Bulletin: Hydrotechnical Considerations for Dam Safety. Regional hydrometric station data homogeneity was determined using the L-moments method developed by Hosking and Wallis.

1.2 Summary of Results

Table 1.1 below summarizes the regional analysis results and compares them with values from the previous modeling work reported March 21, 2017. Derivation of the Regional Analysis results is presented in the sections below.

Table 1.1: Results Summary

Return Period (Years)	Flow (m^3/s)	
	Regional Analysis	March 21, 2017 Modeling
2	13.5	7.5 – 20.0
1000	36.9	27.4 – 46.7
10000	45.0	-
PMP	-	34.5 – 54.9

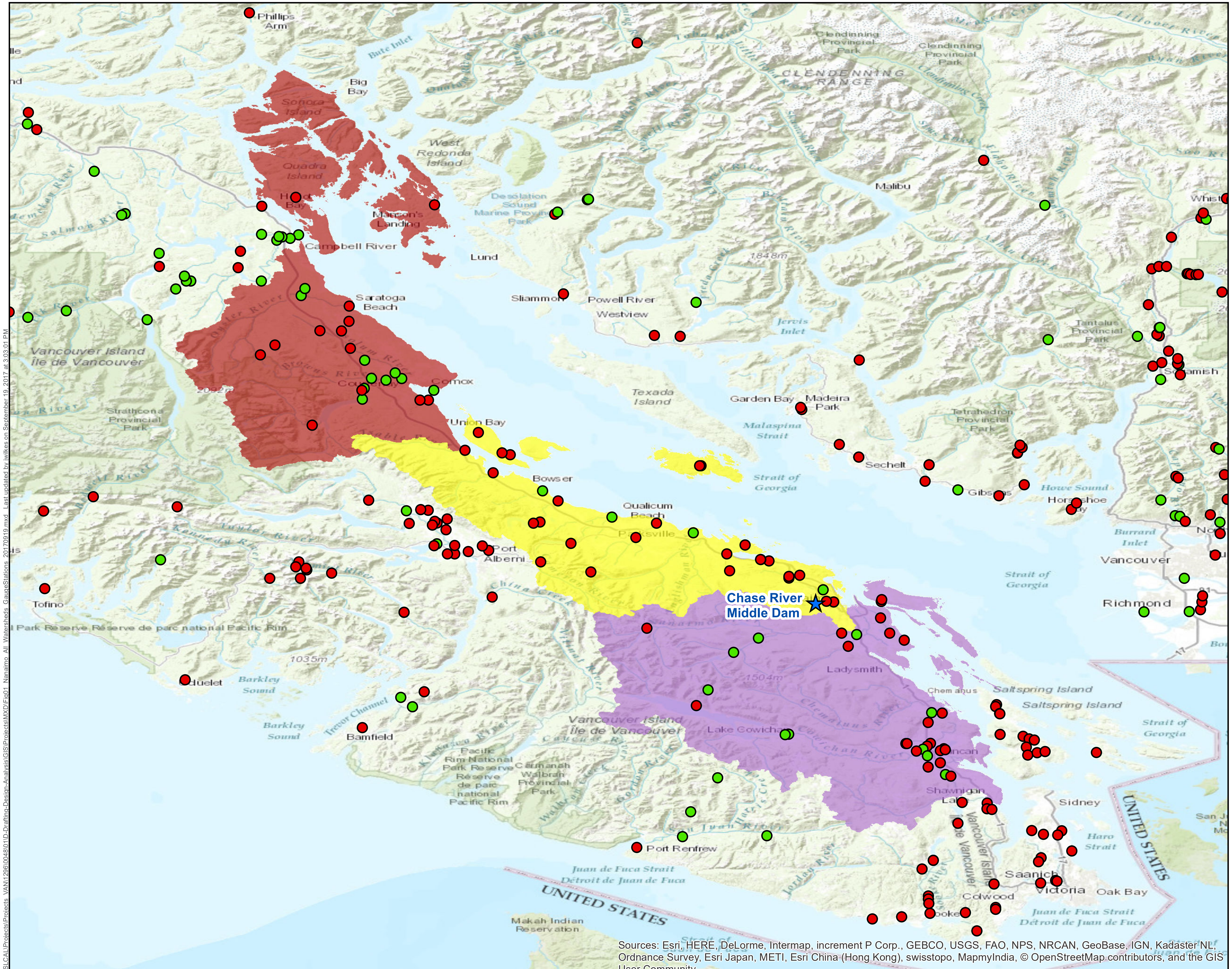
2. Identifying Homogenous Sites

2.1 Initial Review and identifying sites

The first filters to determine which sites have homogenous regional data were location and catchment characteristics. The mountains on Vancouver Island create a hydrologic division between the east and west sides of the island. Weather patterns from the Pacific create significantly increased rainfall along the west side of the island compared to the east. Therefore, stations on only the east side of the island were considered.

To establish the initial set of stations for analysis, hydrologic sub-watersheds were obtained from the iMapBC system[1]. Chase River is located within the 'Parksville' watershed as shown on Figure 1. However, this watershed has a limited number of long-term hydrometric stations on unregulated streams. Therefore, stations from the 'Comox' and 'Cowichan' watersheds were also evaluated to increase the sample size. The watersheds selected for this analysis, and corresponding available stream gauges, are also shown in Figure 1.

[1] Retrieved from http://maps.gov.bc.ca/ess/sv/imapbc/. Watersheds based on the 1:50,000 scale are published by the Ministry of Environment and Climate Change Strategy, Knowledge Management.



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Nanaimo Regional Analysis

Watersheds and All Flow Gauge Stations

Legend

Flow Gauge Stations

- Active
- Discontinued

Watersheds

- COWN
- COMX
- PARK

The accuracy & completeness of information shown on this drawing is not guaranteed. It will be the responsibility of the user of the information shown on this drawing to locate & establish the precise location of all existing information whether shown or not.

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Kilometers

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Coordinate System:
GCS North American 1983

Scale:
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Data Sources:
Data provided by -

Project #: 1296.0048.01
Author: JW
Checked: TS
Status: ~ DRAFT ~
Revision: A
Date: 2017 / 9 / 19

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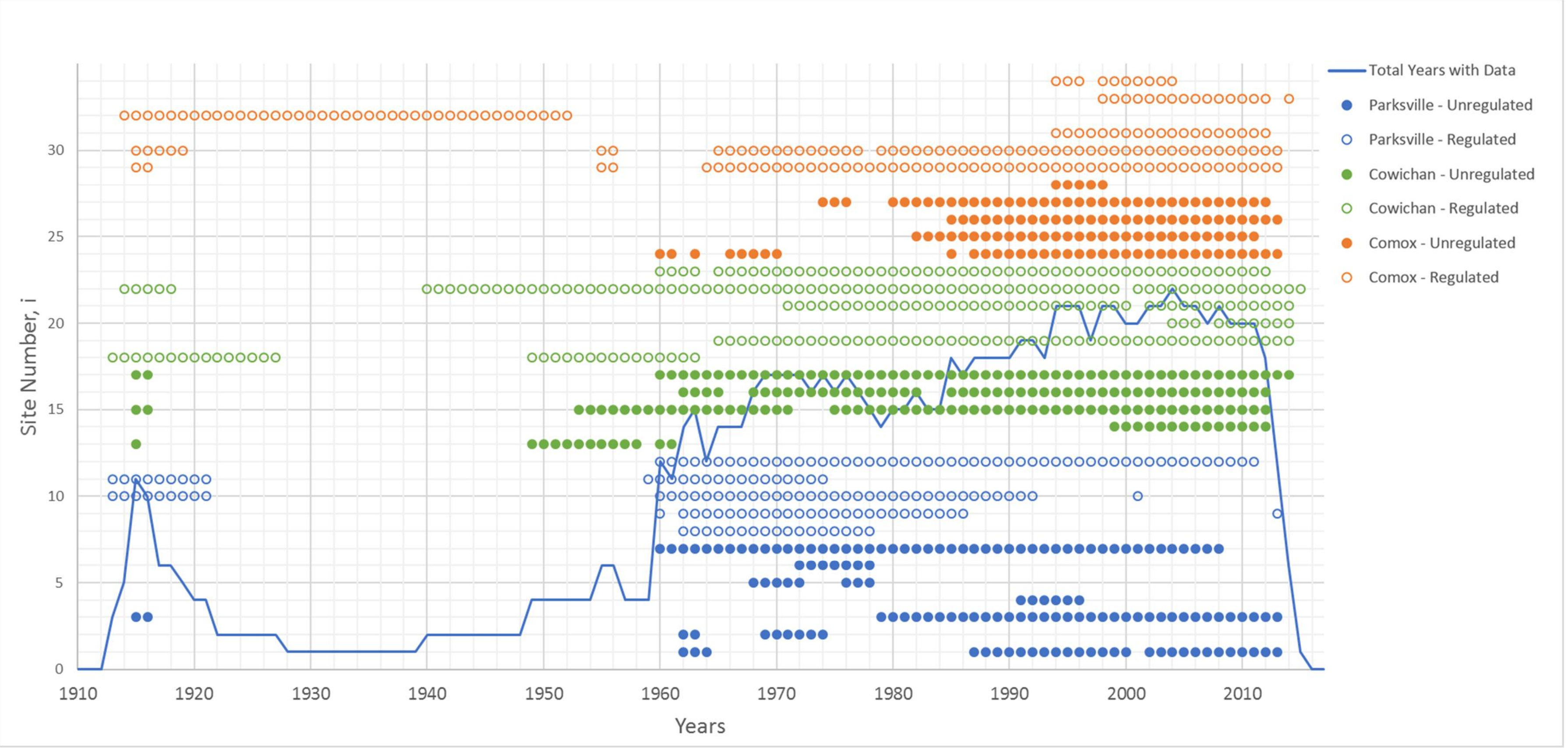
FIGURE 1

The next step in the initial review of stream gauges was to retrieve the Annual Maximum Series (AMS) for each site from Environment Canada. Because some gauges had incomplete or sporadic reporting, the AMS was often shorter than the total number of years of operation. To eliminate the variability associated with extremely small sample sizes, stations with less than 5 years of data were excluded. There were 34 sites in the initial selection. The years of available data from all the sites are summarized in Figure 2 to illustrate what data are available, grouped by watershed, and whether the stream is regulated or not.

Figure 2: Years with Data at Each Site



2.2 Describing Frequency Distributions

The next step in the identification of sites to use for the analysis was to describe the distributions of the AMS series from each site. In principle, all the sites that constitute a homogenous region should be described by the same regional growth curve, with the differences observed between samples due to the sampling variability rather than different stream characteristics (Hosking & Wallis, 1993). Therefore, the first step in describing whether a selection of stream gauges constitute a homogenous region is to describe and compare their sample characteristics.

2.2.1 Moments and Cumulants

There are several different tools used to describe a data set; one of these is by calculating its moments or cumulants, which provide information about the underlying probability distribution that describes the data. The generalized definition for the *n*-th order moment of a real-valued continuous function *f(x)* of a real variable about a value c is:

$$\mu_n = \int_{-\infty}^{\infty} (x-c)^n f(x)\, dx \tag{1}$$

Standard terminology in mathematics usually makes a distinction between *moments* or *raw moments* meaning moments about zero (c = 0) and *central moments* meaning moments about the mean (c = μ_1 or just μ). For n ≥ 3, *normalised moments* are used as well because they are dimensionless and independent of scale; these are also called *normalised n-th central moment* or *standardized moment* and described by the ratio:

$$\frac{\mu_n}{\sigma^n} \tag{2}$$

- The first moment, n = 1, describes the mean of the population, usually denoted as μ.
- The second central moment is known as the *variance* and is denoted as σ^2. Its square root is the standard deviation σ.
- The 3rd normalised moment is known as the *Skewness* and is often denoted using the letter γ. Skewness is a measure of lopsidedness; a symmetrical distribution will have a skewness of zero. A positive skewness indicates that the distribution has a longer tail on the right, and is said to be skewed to the right. A negative skewness has a longer tail on the left, and is said to be skewed to the left.
- The 4th normalised moment is known as the *Kurtosis* and usually denoted by κ. There are several ways of interpreting kurtosis, but it is essentially a measure of the 'tailedness' of a distribution, or how likely an extreme value is to occur relative to a value closer to the median.
- Moments beyond the 4th order are called *High-Order Moments*. Because of the excess degrees of freedom (essentially, raising a value to the exponent n), high-order moments require larger samples to obtain estimates of similar quality. Therefore, the first four moments are the most commonly used, especially for hydrology where the sample sizes are typically small.

The primary difficulty in using moments to describe data sets in a hydrological context is that they are very sensitive to sampling variability due to the large exponents in the calculation. Because hydrological samples such as stream gauges often have very small sample sizes, this approach can make it difficult to differentiate site characteristics (Hosking, 1990). In order to develop a better understanding of the characteristics of hydrologic samples, L-moments are suggested for comparison in place of regular moments.

2.2.2 L-Moments

"L-moments are expectations of certain linear combinations of order statistics." (Hosking, 1990) Although they have been used before, the first unified approach to use them for statistical analysis of probability distributions was developed in (Hosking, 1990). L-moments are analogous to regular moments, but use linear functions of probability weighted moments to avoid the high exponent that regular moments contain. This makes them less prone to the effects of sampling variability than traditional moments. Additionally, there are several characteristics of L-moments that are easier to interpret than traditional moments, such as having defined bounds (for instance, L-skewness is always between -1 and 1).

For a continuous variable X, the generalized definition of an *r*-th order L-moment is:

$$\lambda_r = r^{-1} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} EX_{r-k:r} \tag{3}$$

Where $EX_{i:r}$ is the expected value of the *i*-th largest order statistic of a sample of size *r* selected randomly from X.

(Hosking, 1990) quotes (David, 1981) with the following formula for the expectation of an order statistic for the *j*-th largest value of r values in terms of the QDF:

$$E[X_{j:n}] = \frac{n!}{(j-1)!\,(n-j)!} \int_0^1 x(F) F^{j-1} (1-F)^{n-j}\, \mathrm{d}F \tag{4}$$

Using the definition in equation (3), the first four L-moments can be expanded as follows:

$$\lambda_1 = EX \tag{5}$$

$$\lambda_2 = \frac{EX_{2:2} - EX_{1:2}}{2} \tag{6}$$

$$\lambda_3 = \frac{EX_{3:3} - 2EX_{2:3} + EX_{1:3}}{3} \tag{7}$$

$$\lambda_4 = \frac{EX_{4:4} - 3EX_{3:4} + 3EX_{2:4} - EX_{1:4}}{4} \tag{8}$$

From the expansions above, a few things can be expressed (Hosking & Wallis, 1997):

- The first L-moment is equal to the mean, and therefore provides information about the location of the distribution
- For a sample size of 2, the order statistics are $X_{1:2}$ (smallest) and $X_{2:2}$ (largest). For a highly dispersed distribution, the difference between $X_{1:2}$ and $X_{2:2}$ would be large, and vice versa. The second L-moment therefore contains information about the variability or scale of the distribution.
- For a sample size of 3, the order statistics are $X_{1:3}$ (smallest), $X_{2:3}$ (median), and $X_{3:3}$ (largest). For a negatively skewed distribution, the difference $X_{2:3}$ -$X_{1:3}$ would be larger than the difference between $X_{3:3}$ -$X_{2:3}$ and vice versa, meaning the third L-moment contains information about the skewness of a distribution
- For a sample size of 4, Hosking suggests it is clearer to write it as $(X_{4:4} – X_{1:4}) – 3(X_{3:4} – X_{2:4})$, to illustrate that it measures how much further apart the two extreme values are then the two central values, and thus contains information about kurtosis.

Analogously to regular normalised moments, Hosking has also defined three L-moment ratios that are normalised to describe the shape of a probability distribution independently of scale:

- $\tau = \lambda_2 / \lambda_1$ is known as the L-CV (coefficient of variability)
- $\tau_3 = \lambda_3 / \lambda_2$ is known as the L-Skewness
- $\tau_4 = \lambda_4 / \lambda_2$ is known as the L-kurtosis

These ratios can be used similarly to regular moments to summarize the shape of a probability distribution, but their linear nature means that they are less prone to sampling variability, and therefore better for hydrology, where the sample sizes are often small (Hosking, 1990). In addition, they are “more easily interpretable as measures of distributional shape” (Hosking & Wallis, 1993)

2.2.3 Sample L-Moments

The direct estimators proposed by Q.J. Wang were used in order to estimate the L-Moments of a population from the sample AMS for each site. The following direct estimators are given (Wang, 1996):

$$l_1 = \binom{n}{1}^{-1} \sum_{i=1}^{n} x_{(i)} \tag{9}$$

$$l_2 = \frac{1}{2}\binom{n}{2}^{-1} \sum_{i=1}^{n} \left\{ \binom{i-1}{1} - \binom{n-i}{1} \right\} x_{(i)} \tag{10}$$

$$l_3 = \frac{1}{3}\binom{n}{3}^{-1} \sum_{i=1}^{n} \left\{ \binom{i-1}{2} - 2\binom{i-1}{1}\binom{n-i}{1} + \binom{n-i}{2} \right\} x_{(i)} \tag{11}$$

$$l_4 = \frac{1}{4}\binom{n}{4}^{-1} \sum_{i=1}^{n} \left\{ \binom{i-1}{3} - 3\binom{i-1}{2}\binom{n-i}{1} + 3\binom{i-1}{1}\binom{n-i}{2} - \binom{n-i}{3} \right\} x_{(i)} \tag{12}$$

2.3 Homogeneity Testing

After the initial screening of the samples, all the AMS from Environment Canada were characterised using the techniques described above. Table 1 contains the characteristic L-moment ratios of each of the sample sites that were studied further:

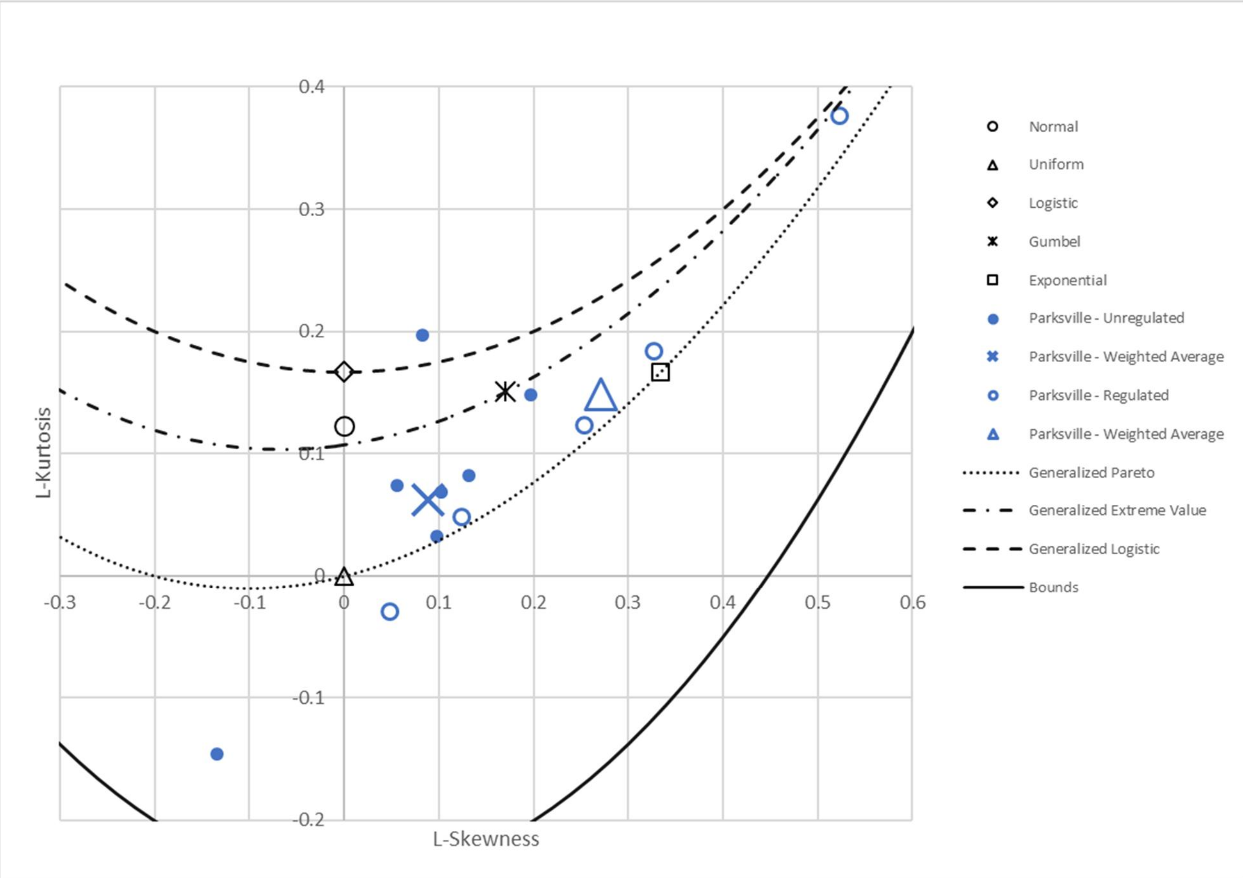
Table 2.1: Initial Pooling Group

Site Number, i	Station ID	Regulated?	Watershed	Number of Years of Data (Sample Size), n	L-CV	L-Skewness	L-Kurtosis
1	08HB032	N	PARK	29	0.24	0.13	0.08
2	08HB005	R	COWN	30	0.25	0.26	0.23
3	08HB034	R	COWN	50	0.23	0.01	0.04
4	08HB027	N	PARK	8	0.13	0.08	0.20
5	08HB003	N	COWN	13	0.23	0.17	0.04
6	08HB092	R	COWN	10	0.15	-0.09	0.22
7	08HB030	R	PARK	17	0.25	0.12	0.05
8	08HB041	R	COWN	43	0.22	-0.02	0.14
9	08HB002	N	PARK	37	0.24	0.06	0.07
10	08HA072	N	COWN	14	0.14	-0.12	0.04
11	08HA001	N	COWN	59	0.22	0.14	0.17
12	08HA002	R	COWN	80	0.17	0.05	0.11
13	08HA016	N	COWN	47	0.26	0.08	0.03
14	08HB080	N	PARK	6	0.14	-0.13	-0.15
15	08HA011	R	COWN	52	0.20	0.04	0.11
16	08HB029	R	PARK	27	0.27	0.05	-0.03
17	08HB004	R	PARK	43	0.30	0.25	0.12
18	08HA003	N	COWN	57	0.20	0.00	0.13
19	08HB001	R	PARK	25	0.36	0.52	0.38
20	08HB022	R	PARK	52	0.33	0.33	0.18
21	08HB037	N	PARK	8	0.24	0.10	0.07
22	08HB045	N	PARK	7	0.28	0.20	0.15
23	08HB024	N	PARK	49	0.25	0.10	0.03
24	08HB011	R	COMX	54	0.22	0.02	0.10
25	08HB006	R	COMX	55	0.21	0.01	0.07
26	08HB034	R	COMX	19	0.20	0.02	0.20
27	08HB007	R	COMX	39	0.19	0.23	0.22
28	08HB025	N	COMX	36	0.24	0.24	0.16
29	08HB074	N	COMX	30	0.26	0.18	0.16
30	08HB075	N	COMX	29	0.17	-0.14	0.05
31	08HB089	R	COMX	16	0.22	0.21	0.17
32	08HD030	R	COMX	10	0.36	0.14	0.20
33	08HD011	N	COMX	36	0.21	0.06	0.12
34	08HD016	N	COMX	5	0.09	-0.24	0.33

When comparing a site's L-moments, Hosking suggests the use of L-moment ratio diagrams that plot the ratios against each other. We did this in two steps, comparing regulated vs unregulated streams, and comparing the three BC MoE watersheds to each other in order to make the figures clearer.

Figure 3 below shows the L-moment ratios for regulated vs unregulated stream gauges in the Parksville watershed:

Figure 3: L-Moment Ratio Diagram - Regulated vs. Unregulated Streams



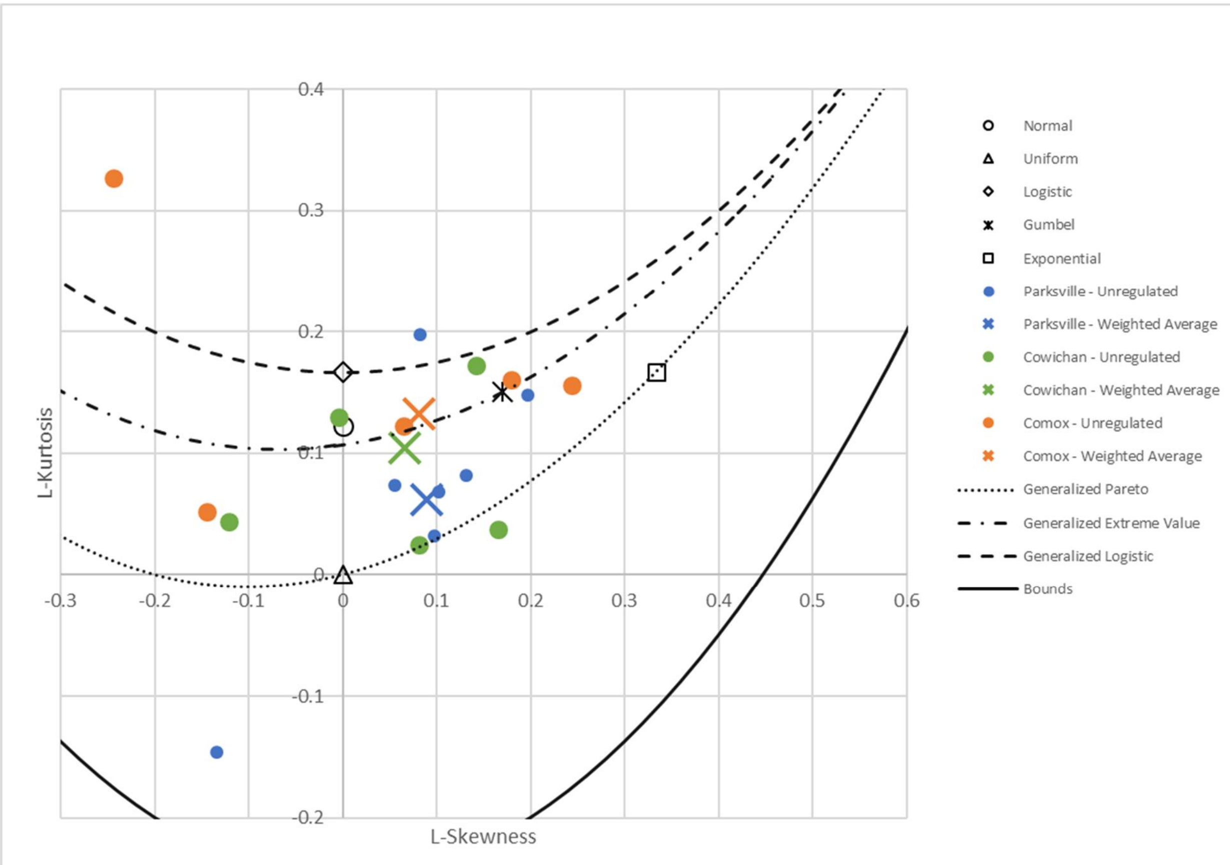
Note the following conclusions drawn from Figure 3:

- The clustering of unregulated sites around the unregulated average values (indicated by the blue X) indicates that most of the unregulated sites are likely homogenous - except 08HB080, which has negative L-skewness and L-Kurtosis. A closer examination shows that this site only has 6 years of data, so this might be the result of large sampling variability.
- The regulated sites show no obvious clustering; this is not surprising because there is no information given about how the sites are controlled. They may simply be controlled using different targets which means their sample characteristics will no longer reflect the original regional characteristics.

- The black lines and data points in the background show how the sample characteristics compare to the characteristics of some known distributions that are often used in frequency analysis (Hosking, 1990)

Based on the scatter evident in this test, and lack of available information about how the regulated streams are controlled, it was decided to eliminate the regulated streams from the regional analysis. Figure 4 below shows all the unregulated stream gauges, coloured according to the MoE watersheds:

Figure 4: L-Moment Ratio Diagram - Comparison of MoE Watersheds



In general, this plot shows that the three watersheds could be treated as a homogenous pooling group, with a few outliers. Although the average L-Kurtosis for the Parksville watershed is slightly lower than the other two, the variability between sites within each watershed is greater than the differences between the average values. Based on this plot, the unregulated streams from all three watersheds were analysed as a single potential pooling group using a discordance test as another measure to assess the similarity between sites.

The discordance test follows the procedure described by Hosking (Hosking & Wallis, 1993). First, each site is described by a matrix called u_i, where:

$$u_1 = \begin{bmatrix} t^{(i)} \\ t_3^{(i)} \\ t_4^{(i)} \end{bmatrix} \tag{13}$$

Then call the unweighted group average $\bar{u}$:

$$\bar{u} = N^{-1} \sum_{i=1}^{N} u_i \tag{14}$$

Then, describe the sample covariance matrix as:

$$S = (N-1)^{-1} \sum_{i=1}^{N} (u_i - \bar{u})(u_i - \bar{u})^T \tag{15}$$

Finally, describe a discordancy measure D_i for each site such that:

$$D_i = \frac{1}{3} (u_i - \bar{u})^T S^{-1} (u_i - \bar{u}) \tag{16}$$

Large values of D_i indicate sites that are most discordant, in other words furthest from the average of all the sites. Hosking and Wallis caution that using the formal significance tests requires knowledge of the sample distribution of the statistic D_i that is not accurately known. They have conducted some simulations assuming that sample L-moments of the data are independent and normally distributed, but use them as guidelines rather than strict decision criteria. In general, they tentatively suggest using $D_i \geq 3$ as a criterion for declaring a site to be unusual (Hosking & Wallis, 1993).

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Calculating the discordancy measure for all the unregulated sites produces the following results:

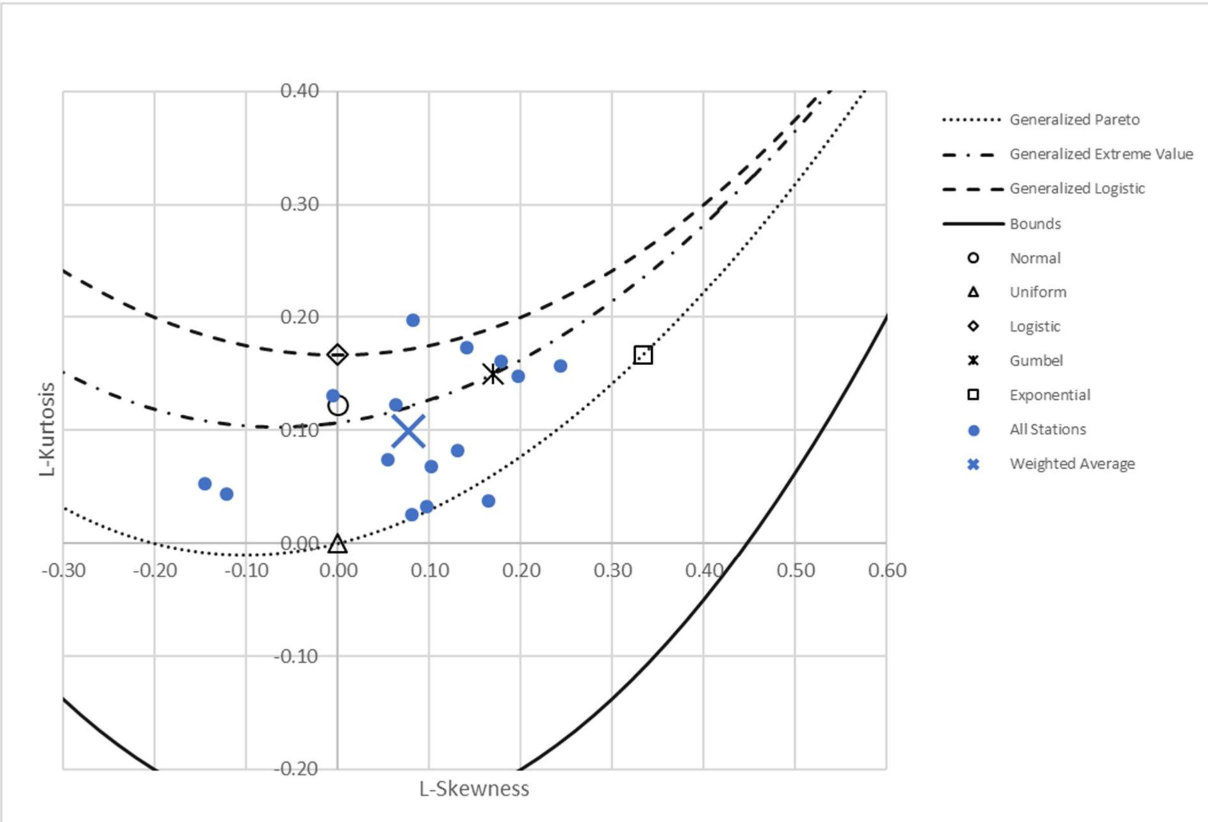
Table 2.2: Discordancy Measure Summary

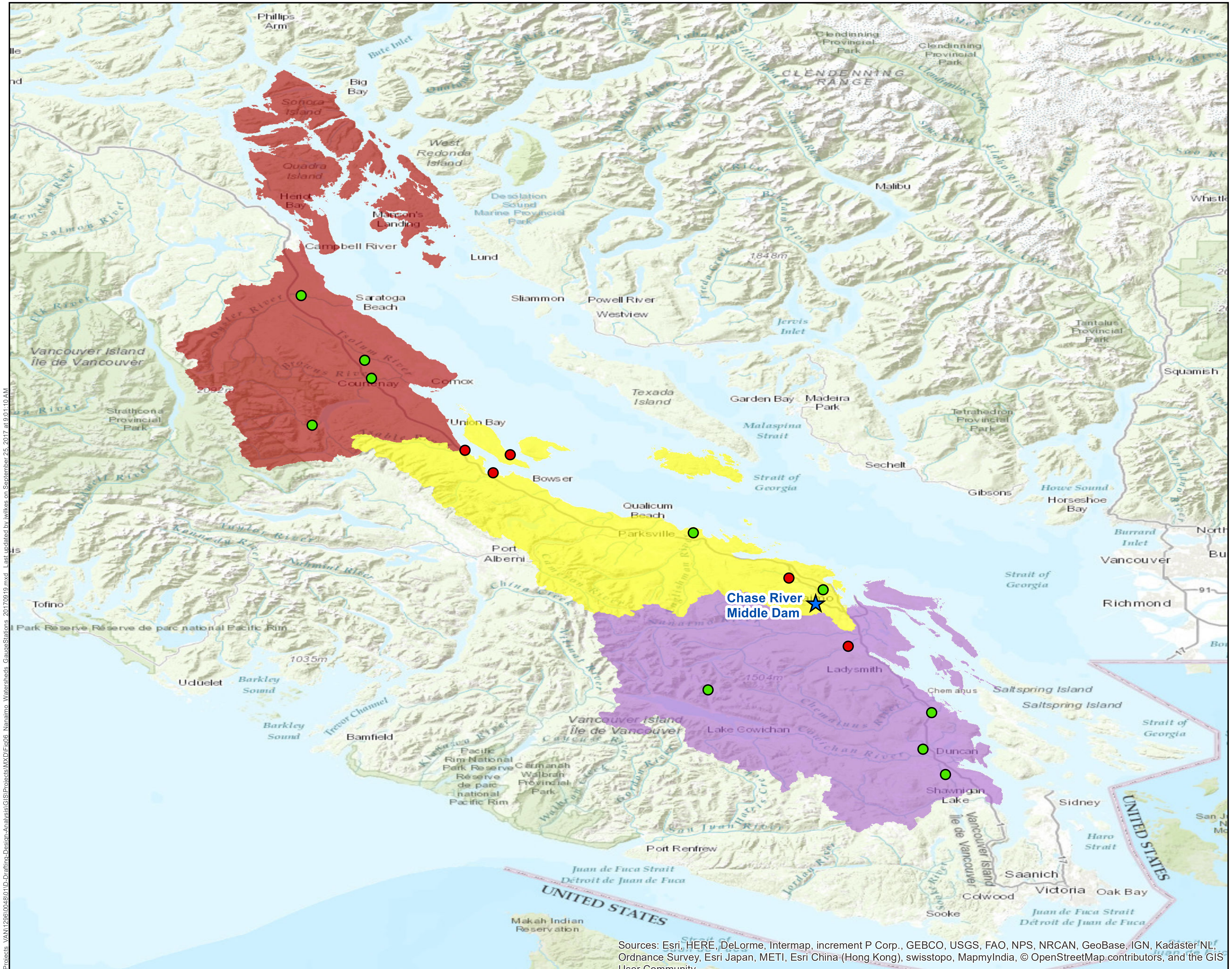
Sample Series Number	Station ID	Regulated ?	Watershed	Number of Years of Data (Sample Size), n	L-CV	L-Skewness	L-Kurtosis	Di
1	08HB032	N	PARK	29	0.24	0.13	0.08	0.15
4	08HB027	N	PARK	8	0.13	0.08	0.20	2.60
9	08HB002	N	PARK	37	0.24	0.06	0.07	0.41
14	**08HB080**	**N**	**PARK**	**6**	**0.14**	**-0.13**	**-0.15**	**3.07**
21	08HB037	N	PARK	8	0.24	0.10	0.07	0.15
22	08HB045	N	PARK	7	0.28	0.20	0.15	0.69
23	08HB024	N	PARK	49	0.25	0.10	0.03	0.34
5	08HB003	N	COWN	13	0.23	0.17	0.04	0.58
10	08HA072	N	COWN	14	0.14	-0.12	0.04	0.74
11	08HA001	N	COWN	59	0.22	0.14	0.17	0.33
13	08HA016	N	COWN	47	0.26	0.08	0.03	0.70
18	08HA003	N	COWN	57	0.20	0.00	0.13	0.13
28	08HB025	N	COMX	36	0.24	0.24	0.16	1.05
29	08HB074	N	COMX	30	0.26	0.18	0.16	0.46
30	08HB075	N	COMX	29	0.17	-0.14	0.05	0.97
33	08HD011	N	COMX	36	0.21	0.06	0.12	0.03
34	**08HD016**	**N**	**COMX**	**5**	**0.09**	**-0.24**	**0.33**	**3.58**
Weighted Average:						0.08	0.10	
Direct Average:				27	0.21	0.05	0.10	

Based on this test, it is suggested that the two sites 08HB080 and 08HD016 are not homogenous with the rest of the group. Both of these sites have very small sample sizes (6 years and 5 years, respectively), meaning that the differences may be due to sampling variability rather than catchment characteristics, but this cannot be determined without further information. For further processing, these sites have been removed from the pooling group because they are considered outliers. The remaining sites which constitute the final pooling group, are shown in Figure 6.

The resulting L-moment diagram with the outliers removed is shown in Figure 5 below:

Figure 5: L-Moment Ratio Diagram - Final Pooling Group





Nanaimo Regional Analysis

Watersheds and Final Pooling Group Flow Gauge Stations

Legend

Flow Gauge Stations

- Active
- Discontinued

Watersheds

- COWN
- COMX
- PARK

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0 10 20 40
Kilometers

N

Coordinate System: GCS North American 1983

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Data Sources:
Data provided by -

Project #:	1296.0048.01
Author:	JW
Checked:	TS
Status:	~ DRAFT ~
Revision:	A
Date:	2017 / 9 / 25

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FIGURE 6

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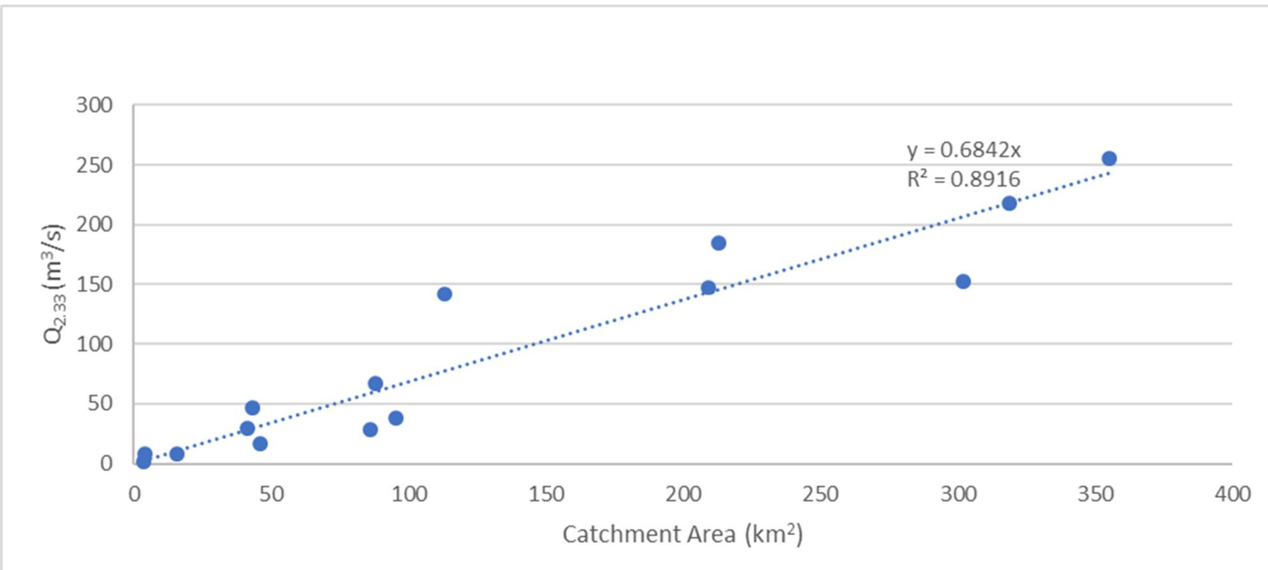
3. Index Flood Method

3.1 Finding the index flood

Based on the L-moment diagram of the pooling group, the closest standard distribution to the average for the selected sites is the Generalized Extreme Value, or GEV. The software program Hyfran+ was used to find the best-fit curve based on the GEV distribution for each of the sites in the pooling group. Based on the best-fit curves, the index flood was calculated for each station as $Q_{2.33}$, or the flow rate corresponding to a 2.33 year return period, which is the probability weighted average of the exceedance curves. The resulting index flood flow rates for each catchment were then graphed against the catchment size to develop a relationship between $Q_{2.33}$ and catchment size:

Figure 7: Index Flood Rate vs. Catchment Area

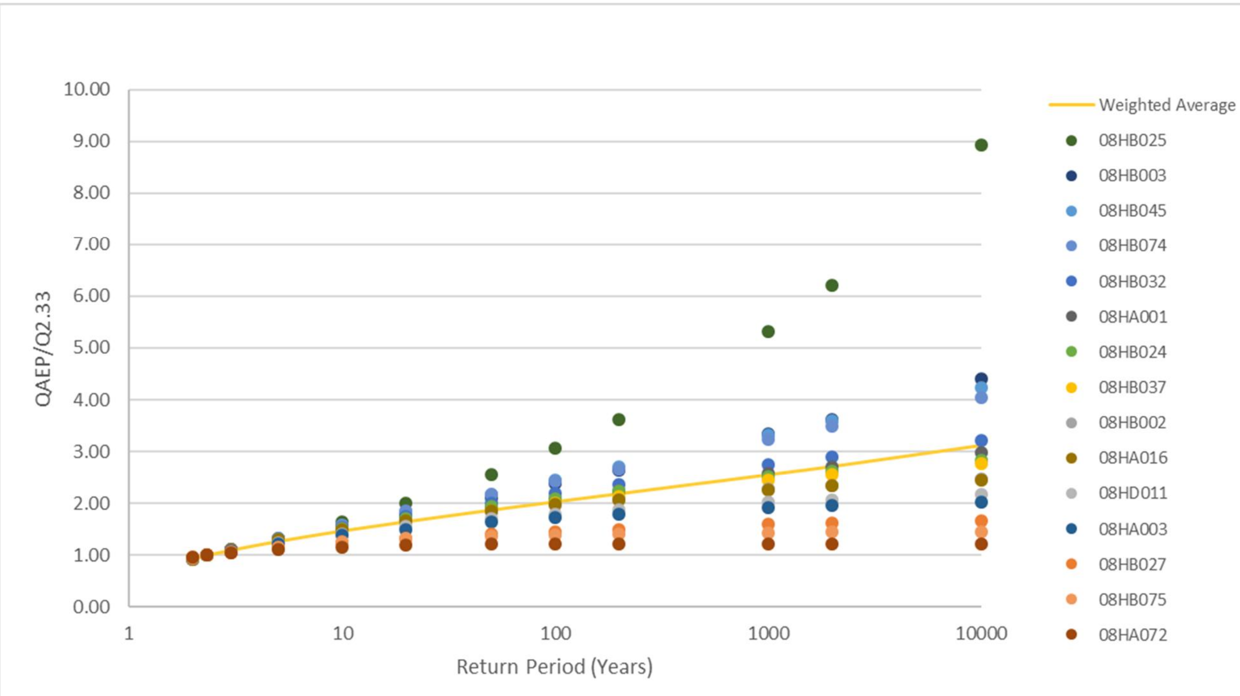


The best-fit curve on this graph shows that the index flood for this pooling group is generally linear, and 0.6842 times the catchment area in km^2. Based on this relationship, the index flood for Chase River, with a catchment area of 21 km^2, should be approximately 14.4 m^3/s.

3.3 Finding the regional growth curve

The best-fit GEV curves for each site are also used to find the regional growth curve, which is the curve that should describe every site in the homogenous region, if sampling variability were removed. To establish this curve, the best-fit curves from each site are normalised by the site's index flood, to establish the dimensionless curve $Q_{AEP}/Q_{2.33}$ for each site. These curves are then averaged, weighting by the number of samples at each site (n_i) because the sampling variability is inversely proportional to sample size. The resulting weighted average curve is the regional growth curve. The dimensionless curves from each station are shown below in Figure 8 along with the weighted average, which is the regional growth curve:

Figure 8: Normalized Best-Fit Curves



It appears that there is one station that is an outlier, 08HB025. This can also be seen in the discordancy test in section 2.3 above, where its D_i value is higher than average (although not high enough to be considered discordant). Examining its AMS shows that there are two years with very large flow rates that caused the GEV best-fit curve to jump. However, this is not necessarily an indicator of any problem; the largest peak flow occurred in October 1968, and appears to have recorded correctly (i.e. there is no evidence of a data jump or gap in the continuous series). Therefore, this station was maintained in the area weighting.

The final step in the index-flood method is to multiple the dimensionless regional growth curve with the index flood for Chase River calculated in section 3.1 above, 14.4 m^3/s. The resulting flow vs exceedance curve is tabulated below:

Table 3.1: Chase River Flow Rates

Return Period (Years)	Flow Rate (m^3/s)
10000	45.0
2000	39.2
1000	36.9
200	31.6
100	29.3
50	27.0
20	23.8
10	21.2
5	18.3
3	15.8
2.33	**14.4**
2	13.5

4. Conclusions

The flow vs exceedance curve calculated here for Chase River is comparable with the previous model of the Chase River system that was created by USL (March 21, 2017). From the previous model, the 1:2 year flow was estimated to be between 7.5 m^3/s and 20.0 m^3/s (depending on the duration of the rainfall event), compared with 13.5 m^3/s estimated using the regional comparison methodology outlined here. Similarly, the 1:1000 year flow was previously estimated to be between 27.4 m^3/s and 46.7 m^3/s, again dependant on the duration of the rainfall event, and here it is estimated as 36.9 m^3/s. The PMP was estimated by Golder Associates (2014) to be close to a return period of 1:50:000, which is beyond the current extrapolation; however, its previously estimated flow rate of 34.5 m^3/s to 54.9 m^3/s is generally comparable with the 45.0 m^3/s estimated here for the 1:10000 return period.

Sincerely,

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PROFESSIONAL PROVINCE OF A. G. ZACHARY BRITISH COLUMBIA ENGINEER

2017-09-26

TSwailes

Taylor Swailes, E.I.T.
Water Resources

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